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A FREE RELATIVISTIC ANYON WITH CANONICAL SPIN ALGEBRA

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Abstract

We discuss a relativistic free particle with fractional spin in 2+1 dimensions, where the dual spin components satisfy the canonical angular momentum algebra $\{S_\mu, S_\nu\} = \epsilon_{\mu\nu\gamma}S^\gamma$. It is shown that it is a general consequence of these features that the Poincarè invariance is broken down to the Lorentz one, so indicating that it is not possible to keep simultaneously the free nature of the anyon and the translational invariance.

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1 Introduction

In the last few years the study of particles with generalized statistics, the so called anyons [1], has been a growing field of interest. This is mainly due to the possibility of explaining very important physical phenomena, the fractional quantum Hall effect and the high temperature superconductors [2, 3].

However, most of the work on this matter was done through nonrelativistic models, or by using the statistical field A_μ of the topological Chern-Simons term [4]. Notwithstanding, some authors have been looking for the answer to the question whether this statistical field not only changes the statistics, but also endows the anyons with interaction. One has been looking in particular, into the possibility of constructing a point-particle model for anyons, without the need of using any statistical field [5, 6, 7, 8]. This quest should necessarily begin by studying the representations of the Lorentz group in 2+1 dimensions [5, 6, 7].

The Lorentz group in 2+1 dimensions corresponds to the SO(2,1) group whose irreducible unitary representations, are all labeled by a set of half-integer and integer numbers [5]. Thus if we are interested in a Lorentz covariant description of particles with fractional spin, we have to deal in principle with an infinite dimensional representation of the Lorentz generators. On the other hand, Jackiw and Nair have shown in [6] that the anyon posses just one physical polarization state for a given sign of the energy, and they have obtained an induced representation for the Poincarè group on those physical polarization states. What is actually important for us is that, in the induced representation, the spin components commute ($[S_\mu, S_\nu] = 0$) and therefore do not satisfy the usual angular momentum algebra. Nevertheless, some point-particle models corresponding to the induced representation have appeared in the literature [7, 8, 9]. However, as far as we know, none of the models discussed up to now has been able to get two ingredients holding simultaneously, namely the free nature of the relativistic anyon and the canonical spin algebra $\{S_\mu, S_\nu\} = \epsilon_{\mu\nu\gamma} S^\gamma$ for the spin components [8]. In this work we intend to explore the consequences of imposing these features simultaneously. In the next section we show explicitly how this can be carried through in a particular point-particle model. In section three, we perform a model independent analysis, then in section four we draw some conclusions and summarize the work.

2 A model for a free particle with fractional spin

The Poincarè algebra in 2+1 dimensions is given by:

$$\begin{aligned}
[J_\mu, J_\nu] &= \epsilon_{\mu\nu\gamma} J^\gamma \\
[J_\mu, P_\nu] &= \epsilon_{\mu\nu\gamma} P^\gamma \\
[P_\mu, P_\nu] &= 0,
\end{aligned} \tag{1}$$

where J_μ are the dual components of the total angular momentum tensor, *i.e.* $J_\mu = \epsilon_{\mu\nu\gamma} J^{\nu\gamma}$ and we are using $\eta_{\mu\nu} = diag\,(-,+,+)$, $\epsilon_{012} = +1$.

Let us suppose now that we have a relativistic model for a point-particle whose total angular momentum tensor and momenta components are given in the phase space by

$$J_{\mu\nu} = x_\mu p_\nu - p_\mu x_\nu + n_\mu \pi_\nu - \pi_\mu n_\nu\,,\; P_\mu = p_\mu. \tag{2}$$

Assuming that the vectors $x_\mu, p_\mu, n_\mu, \pi_\mu$ satisfy the canonical Poisson brackets, $\{x_\mu, p_\nu\} = \eta_{\mu\nu} = \{n_\mu, \pi_\nu\}$, with the remaining brackets vanishing, it is easy to derive the Poincarè algebra in terms Poisson brackets. Note that the variables (n_μ, π_μ) are just introduced to describe the spin of the particle $S_\mu = \epsilon_{\mu\nu\beta} n^\nu \pi^\beta$, which satisfies $\{S_\mu, S_\nu\} = \epsilon_{\mu\nu\gamma} S^\gamma$. The Poincarè algebra (1) possess two quadratic Casimir invariants which can be represented in our model by p^2 and $S \cdot p$, they will be used in the form of constraints to specify the mass and helicity of the particle respectively:

$$\phi_1 = p^2 + m^2 \approx 0 \tag{3}$$
$$\phi_2 = S \cdot p + \alpha m \approx 0 \quad , \tag{4}$$

where α is an arbitrary real constant. In analogous fashion to the $\alpha = 1/2$ case [6], Eq.(4) will play the role (for arbitrary α) of the Dirac equation [10, 6]. We expect that the constraints ϕ_1, ϕ_2 be first class, in order to specify the physical states which belong to the Poincarè representation specified by α and m, such that the hamiltonian of the model can be assumed to be of the form

$$H \;=\; e\,\phi_1 \,+\, \sigma\,\phi_2 \,+\, \lambda^a\chi_a, \tag{5}$$

where e, σ and λ^a ($a=1,...,4$) are the lagrange multipliers, whereas χ_a represent four second class constraints. The reason why we need exactly four second class constraints comes from a simple counting of degrees of freedom as follows; by assuming that the spin components S_μ and the momenta p_μ are parallel, one can show for arbitrary α that $g = 2$, where g is the gyromagnetic constant [11]. This result agrees with previous calculations [12] in field theory. If this is indeed the case, the spin of a particle in 2+1 dimensions is entirely specified by the helicity $S \cdot p$ which is fixed in our model by the constraint ϕ_2 and since ϕ_2 is assumed to be a first class constraint, it will eliminate two degrees of freedom of the extra six variables (n_μ, π_μ) introduced to describe the spin degrees of freedom. So we have to further impose four second class constraints in order to eliminate the remaining variables. It can be checked that the easiest way to assure that S_μ and p_μ be parallel is to impose the constraints [8, 9]

$$\chi_1 \;=\; p \cdot n \;=\; 0\,,\;\; \chi_2 \;=\; p \cdot \pi \;=\; 0 \quad , \tag{6}$$

these constraints imply

$$S_\mu \;=\; \frac{(S \cdot p)}{p^2}\, p_\mu \quad . \tag{7}$$

Due to the fact that (6) does not depend on x_μ, after quantization the commutation relations of p_μ would not be changed by such constraints and we would have $[S_\mu, S_\nu] \;=\; 0$ as a consequence of (7). In order to recover the usual spin algebra one must further impose x_μ dependent constraints, for instance,

$$\chi_3 \;=\; x \cdot n \;=\; 0,\;\; \chi_4 \;=\; x \cdot \pi \;=\; 0 \quad . \tag{8}$$

The constraints (6) and (8) form a set of four independent second class constraints which can be eliminated by a Dirac bracket $\{\,,\,\}_D$ leading to

$$\begin{aligned} \{\pi_\mu, \pi_\nu\}_D &\;=\; 0 \;=\; \{n_\mu, n_\nu\}_D \;, \\ \{n_\mu, \pi_\nu\}_D &\;=\; \eta_{\mu\nu} \end{aligned}$$

$$
\begin{aligned}
\{p_\mu, n_\nu\}_D &= \frac{[(\pi\cdot n)n_\mu \,-\, n^2\pi_\mu]\,p_\nu}{S^2}\,, \\
\{p_\mu, \pi_\nu\}_D &= \frac{[\pi^2 n_\mu \,-\, (\pi\cdot n)\pi_\mu]\,p_\nu}{S^2}\,, \\
\{x_\mu, \pi_\nu\}_D &= \frac{[\pi^2 n_\mu \,-\, (\pi\cdot n)\pi_\mu]\,x_\nu}{S^2}\,, \\
\{x_\mu, x_\nu\}_D &= \frac{x^2}{S^2}\,\epsilon_{\mu\nu\gamma}S^\gamma\,, \\
\{p_\mu, p_\nu\}_D &= \frac{p^2}{S^2}\,\epsilon_{\mu\nu\gamma}S^\gamma\,, \\
\{x_\mu, p_\nu\}_D &= \frac{[(x\cdot p)\epsilon_{\mu\nu\gamma}S^\gamma \,+\, S_\mu S_\nu]}{S^2}.
\end{aligned} \tag{9}
$$

Although rather complicated, the above brackets have some remarkable features. Firstly, the Dirac brackets involving only the extra variables (n_μ, π_μ) are exactly equal to the canonical Poisson brackets, therefore we recover the canonical spin algebra that we were searching, *i. e.*, one checks that

$$
\{S_\mu, S_\nu\}_D \,=\, \epsilon_{\mu\nu\gamma}S^\gamma. \tag{10}
$$

Furthermore, using (9) we deduce (for future comparison) :

$$
\{x_\mu, S_\nu\}_D \,=\, \epsilon_{\mu\nu\gamma}x^\gamma \;\;,\;\; \{p_\mu, S_\nu\}_D \,=\, \epsilon_{\mu\nu\gamma}p^\gamma. \tag{11}
$$

By comparing (11) with the Poincarè algebra (1) we notice that S_μ behave like the total angular momentum J_μ and (11) is nothing but the Lorentz transformation law for the vectors x_μ and p_μ. Indeed, since the constraints (6) imply $\epsilon_{\mu\nu\gamma}S^\nu p^\gamma \,=\, 0$, we analogously have from (8) that $\epsilon_{\mu\nu\gamma}S^\nu x^\gamma \,=\, 0$ and therefore the canonical angular momentum $L_\mu \,=\, \epsilon_{\mu\nu\gamma}x^\nu p^\gamma$ vanishes and we can identify S_μ with J_μ. Thus, the brackets (10), (11) and the fact that $\{p_\mu, p_\nu\}_D \,\neq\, 0$ (see (9)) are simply telling us that the Poincarè algebra (1) has been broken down to the Lorentz algebra due to the lack of translational invariance of the constraints (8). In the next section we will show that the identification between S_μ and J_μ and the absence of translational invariance are not peculiar features of this model but a consequence of the spin algebra and the parallelness of S_μ and p_μ. Another important characteristic of the brackets (9) to be noticed, is that they guarantee the conservation of p_μ and S_μ $\left(\dot{p}_\mu \,=\, \{p_\mu, H\}_D \,=\, 0 \,=\, \dot{S}_\mu\right)$ as well as their gauge invariance. These

conservation laws can be used to show that $\ddot{x}_\mu = 0$ and therefore the model under study really corresponds to a free particle, although the Dirac brackets involving x_μ and p_μ are quite complicated. So, we were able to reach the goal of obtaining a free relativistic anyon with canonical spin algebra.

3 A model independent analysis

In this section, we will work from a more general point of view without specifying how the phase space (x_μ, p_ν) has to be extended to include extra variables to describe the spin degree of freedom, and that is precisely what we mean by a model independent analysis. Let us take our hamiltonian to be of the form (5) but, since we are not going to specify the extended phase space, we have, instead of four, N second class constraints χ^A (A = 1,...,N) where N will of course depend on the number of variables of the extended phase space. After the elimination of the constraints χ^A by some Dirac bracket[1] $\{\,,\,\}^*$, the hamiltonian becomes simply

$$H = e\left(p^2 + m^2\right) + \sigma\left(S \cdot p + \alpha\, m\right). \tag{12}$$

Now we would like to examine the equations of motion which come from H. For a generic phase space variable q we have,

$$\dot{q} = \{q, H\}^* \equiv \{q, H\} - f_{AB}\left\{q, \chi^A\right\}\left\{\chi^B, H\right\} \tag{13}$$

where f_{AB} is some suitable function of the phase space variables which certainly depends on specific features of the extended phase space (see footnote).By using the fact that in general we have $\{\chi^A, \phi_i\} = c_i{}^{Aj}\phi_j + d_{iB}{}^A\chi^B$, where the functions $c_i{}^{Aj}, d_{iB}{}^A$ might depend on the details of the model studied, it is not difficult to derive that, in a typical gauge where e and σ are set to a constant,

$$\ddot{x}_\mu = \sigma^2\epsilon_{\mu\nu\gamma}p^\nu S^\gamma + h^i_\mu\phi_i \tag{14}$$

where h^i_μ are functions of the phase space variables which certainly depend on specific features of the extended phase space. Since $\varphi_\mu = \epsilon_{\mu\nu\gamma}p^\nu S^\gamma$ are clearly independent of the first class constraints ϕ_i we come to the important conclusion that, independent of the specific details of the model, p^μ and S^μ

[1] In this section we relax the definition of the Dirac bracket in order to include brackets used to eliminate second class constraints which are not linearly independent.

must be parallel, i.e. $\varphi_\mu = 0$, in order that the particle be free ($\ddot{x}_\mu = 0$) and besides, we have to assume that the model proposed is such that the second term on the r.h.s. of (14) must independently vanish.[2] Since $\phi_i\,(i = 1, 2)$ are the only first class constraints of the theory the parallelness of p_μ and S_μ must be a consequence of some of the second class constraints χ^A. Thus, we can use from now on the identification (7) strongly. Furthermore,if we now impose that the particle besides being free possess spin components S_μ which are gauge invariant[3] ,i.e. $\{S_\mu, \phi_i\}^* = 0$ and satisfy the canonical spin algebra,

$$\{S_\mu, S_\nu\}^* = \epsilon_{\mu\nu\gamma} S^\gamma \quad , \tag{15}$$

we consequently have from (7) that

$$\{S_\mu, p_\nu\}^* = \epsilon_{\mu\nu\gamma} p^\gamma \quad . \tag{16}$$

From (16) and (7) we obtain

$$\{p_\mu, p_\nu\}^* = \frac{p^2}{S^2} \epsilon_{\mu\nu\gamma} S^\gamma \quad , \tag{17}$$

now by using that p_μ is a vector, i.e. $\{p_\mu, J_\nu\}^* = \epsilon_{\mu\nu\gamma} p^\gamma$ where $J_\mu = \epsilon_{\mu\nu\gamma} x^\nu p^\gamma + S_\mu$, we can deduce the following expression

$$\epsilon_{\nu\gamma\beta} p^\beta \{p_\mu, x^\gamma\}^* \;\; \frac{p^2}{S^2} \left((S\cdot x)\eta_{\mu\nu} - x_\mu S_\nu\right) \tag{18}$$

from which we have $x_\mu = (S \cdot x / S \cdot p)\, p_\mu$ and thus $L_\mu = \epsilon_{\mu\nu\gamma} x^\nu p^\gamma = 0$ identically, consequently J_μ and S_μ can be identified. Therefore it seems that the only way out for the spin components of a free fractional spinning particle to satisfy the usual angular momentum algebra is to identify them with the total angular momentum J_μ which will certainly satisfy such algebra. Thus, we have seen that the identification between S_μ and J_μ as well as the lack of translational invariance (see (17)) are not special features of the last section model but a more general consequence of the canonical spin algebra, Lorentz covariance and the requirement that the particle be free. Besides

[2] The reader can check that in order to have $h^i_\mu = 0$ it is sufficient, although not necessary, that ${c_i}^{Aj} = 0$.

[3] Note that this is a more severe requirement than to simply impose that S_μ be observables.

the above general features, we can also almost fully determine $\{x_\mu, x_\nu\}^*$ and $\{x_\mu, p_\nu\}^*$ from our basic assumptions as follows. First notice that from the identification between S_μ and J_μ we have for the vector x_μ:

$$\{x_\mu, S_\nu\}^* = \epsilon_{\mu\nu\gamma} S^\gamma, \tag{19}$$

since x_μ and p_μ are parallel like p_μ and S_μ we can write $x_\mu = (S \cdot x/S^2)\, S_\mu$, thus by using (19) and the spin algebra (15) we obtain (compare with (9))

$$\{x_\mu, x_\nu\}^* = \frac{x^2}{S^2} \epsilon_{\mu\nu\gamma} S^\gamma \quad . \tag{20}$$

In order to fix $\{x_\mu, p_\nu\}^*$ we have to work a little more. By use of the equality $\{x_\gamma, (S \cdot p) S_\mu\}^* = \{x_\gamma, S^2 p_\mu\}^*$ one can deduce

$$\{x_\gamma, p_\nu\}^* \mathcal{P}^\nu_\mu = \left(\frac{S \cdot p}{S^2}\right) \epsilon_{\gamma\mu\beta} x^\beta \quad , \tag{21}$$

where $\mathcal{P}^\nu_\mu = \delta^\nu_\mu - S^\nu S_\mu / S^2$ is a projection operator. Since $\{x_\mu, p_\nu\}^*$ is a second rank tensor with 9 components, we can always decompose it in terms of a 3 components vector c_μ and a 6 components symmetric tensor $N_{\alpha\beta}$ without loss of generality as follows

$$\{x_\gamma, p_\nu\}^* = \epsilon_{\gamma\nu\mu} c^\mu + N_{\gamma\nu} \quad . \tag{22}$$

The quantities c^μ and $N_{\gamma\nu}$ can be partially determined from (21) and the parallelness of x^μ, p^μ and S^μ such that we end up, after some algebra, with

$$\{x_\mu, p_\nu\}^* = \frac{(p \cdot x)}{S^2} \epsilon_{\mu\nu\gamma} S^\gamma + \frac{2\, b_\mu S_\nu}{S^2} - \frac{(S \cdot b) S_\mu S_\nu}{(S^2)^2} \tag{23}$$

where $b_\mu \equiv N_{\mu\nu} S^\nu$.

Since $S_\nu \mathcal{P}^\nu_\mu = 0$, we will have no further informations about b_μ from Eq. (21), but substituting (23) into (18) we obtain $\epsilon_{\mu\nu\gamma} p^\nu b^\gamma = 0$ and therefore $b_\gamma = f\, S_\gamma$ where f is an arbitrary function of the extended phase space variables. Back in (23) we get

$$\{x_\mu, p_\nu\}^* = \frac{(p \cdot x)}{S^2} \epsilon_{\mu\nu\gamma} S^\gamma + f\, \frac{S_\mu S_\nu}{S^2} \quad . \tag{24}$$

Although we cannot fully determine the function f we can obtain further constraints on it. Namely, all Jacobi identities involving x_μ, p_ν and S_γ are

satisfied if and only if $\{f, S_\gamma\}^* = 0$, as can be explicitly checked. Moreover in order that the equations of motion for x_μ be reproduced it is necessary that $f = 1$ on shell. Since there are many candidates for f which satisfy those constraints like, *e.g.*, $f = S \cdot p/\alpha m\,;\, p^2/m^2$; etc.. . We think that the specific form of f depends on the specific model analysed, i.e. depends on how the phase space has been extended.

After having obtained (20),(24) and comparing with the results of the model presented in section two, we see that all the brackets involving x_μ, p_μ and S_γ completely agree if we take $f = 1$ which is the simplest solution to our constraints. Therefore we are now able to recognize that the complicated brackets (9) are not just an unpleasant feature of that specific model but a quite general characteristic of the particles with fractional spin whose spin components satisfy the canonical algebra. It is also clear that such complications will go through the quantization of the model.

We conclude that for a free fractional spinning particle with the canonical spin algebra, the constraints of the theory must guarantee that $L_\mu = 0$ and $S_\mu \propto p_\mu$, and in this case we willl be unavoidably faced with the rather complicated phase space structure given in (17),(20), and (24).

In finishing this section we give a simpler way to derive such results, although less general, where we implement the above physical conditions, i.e. the parallelness of S_μ, p_μ and x_μ using the constraints below

$$\begin{aligned} \varphi_\mu &= \epsilon_{\mu\nu\gamma} p^\nu S^\gamma = 0, \\ \tilde{\varphi}_\mu &= \epsilon_{\mu\nu\gamma} x^\nu S^\gamma = 0 \quad . \end{aligned} \tag{25}$$

It is easy to show that those constraints are all second class, thus they could in principle be eliminated by a Dirac bracket, however this is not possible [13] since the matrix of the Poisson brackets between the constraints has no inverse due to the fact that not all constraints φ_μ and $\tilde{\varphi}_\mu$ are linearly independent. Nevertheless we can eliminate them iteractively by defining a kind of Dirac bracket as follows: [13] first we eliminate, *e.g.*, φ_μ through the bracket $\{\, ,\, \}^\dagger$,

$$\{A, B\}^\dagger = \{A, B\} + \frac{p_\gamma}{p^2 (S \cdot p)} \epsilon^{\gamma\delta\beta} \{A, \varphi_\delta, \} \{\varphi_\beta, B\}\,, \tag{26}$$

and finally we eliminate $\tilde{\varphi}_\mu$ through $\{\, ,\, \}^*$:

$$\{A, B\}^* = \{A, B\}^\dagger + \frac{p^2}{S^2(S \cdot p)} p_\gamma \epsilon_{\mu\nu\gamma} \{A, \tilde{\varphi}_\mu\}^\dagger \{\tilde{\varphi}_\nu, B\}^\dagger . \tag{27}$$

The reader can check that after the elimination of φ_μ and $\tilde{\varphi}_\mu$, the brackets involving S_μ, p_μ and x_μ will match with the general results obtained in this section for the case $f = 1$.

4 Conclusion and Summary

Based on some mild assumptions like Lorentz covariance and the fact that a fractional spinning particle in $2+1$ dimensions obeys, analogous to the spin 1/2 and spin 1 particles [6], an equation of the form $S \cdot P + \alpha m = 0$, we have shown in this work that some peculiar features of some point-particle models recently proposed in the literature can be understood from a rather general point of view. The first of such features is that, irrespective of the spin algebra, we can only have a free ($\ddot{x}_\mu = 0$) fractional spinning particle whenever spin and momentum are parallel, as an example of such rule we can take two models defined in [8]; for the first model, defined in eqs. (2.12)-(2.14) of [8], S_μ and p_μ are parallel whereas this is not the case for the second one, worked out in the fourth section of the same reference, and indeed we have respectively $\ddot{x}_\mu = 0$ and $\ddot{x}_\mu \neq 0$. Furthermore, if the particle besides of being free is required to have spin components which are gauge invariant and satisfy the canonical angular momentum algebra, both in the strong sense, then the algebra of the quantities S_μ, p_ν, and the coordinates x_μ can be, as we have shown, almost completely fixed independently of the details of the extended phase space introduced to describe the spin degree of freedom. It has been shown in particular that our basic assumptions lead to theories where the Poincarè symmetry is broken down to the Lorentz one and the spin components will necessarily play the role of the Lorentz generators. These general predictions have been confirmed in an example given in the first section of this article. Moreover we have also seen that the canonical structure of the variables x_μ and p_ν will be quite complicated which will clearly pose some difficulties in the quantization program.

Finally, it should be noticed that it is no yet clear in the literature (see [6] and comments in [7] and references therein) whether fractional spinning particles in $2+1$ dimensions can be really formulated as free relativistic field

theories and, although our assumptions can be softened, we think that the rather complicated canonical structure and the lack of translational invariance found here can be seen as indications of this fact.

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References

[1] F. Wilczek, in Fractional Statistics and Anyon Superconductivity (World Scientific, Singapore, 1990).

[2] R. B. Laughlin, Phys. Rev. Lett. **50** (1983) 1395; D. P. Arovas, in Geometric Phases in Physics, edited by A. Shapere and F. Wilczek (World Scientific, Singapore, 1989).

[3] J. M. Leinaas and J. Myrheim, Nuovo Cimento **B37** (1977) 1; F. Wilczek, Phys. Rev. Lett. **48** (1982) 1144; **49** (1982) 957.

[4] D. P. Arovas, R. Schrieffer, F. Wilczek and A. Zee, Nucl. Phys. **B251** (1985) 117.

[5] Y. Ohnuki, in Proceedings of 2^{nd} Winter School on Mathematical Physics, Mt. Sorak, Korea, edited by Y. M. Cho (World Scientific, Singapore, 1989).

[6] R. Jackiw and V. P. Nair, Phys. Rev. D **43** (1991) 1933.

[7] S. Forte, Rev. Mod. Phys. **64** (1992) 193.

[8] M. S. Plyushchay, Int. J. Mod. Phys. **A7** (1992) 7045.

[9] M. Chaichian, R. Gonzalez Felipe and D. Louis Martinez, Phys. Rev. Lett. **71** (1993) 3405; see also the comments on this paper: D. Dalmazi and A. de Souza Dutra, hep-th/9402059 preprint; R. Jackiw and V. P. Nair, hep-th/9403010 preprint.

[10] D. Volkov, D. Sorokin and V. Tkach, in Problems in Modern Quantum Field Theory, edited by A. Belavin, A. Klimyk and A. Zamolodchikov (Springer, Berlin, 1989).

[11] C. Chou, V. P. Nair and A. P. Polychronakos, Phys. Lett. **B304** (1993) 105.

[12] J. L. Cortèz, J. Gamboa and L. Velázquez, Phys. Lett. **B286** (1992) 105; Nucl. Phys. **B392** (1993) 645.

[13] A. Dresse, J. Fisch, M. Henneaux and C. Schomblond, Phys. Lett. **B210** (1988) 141.